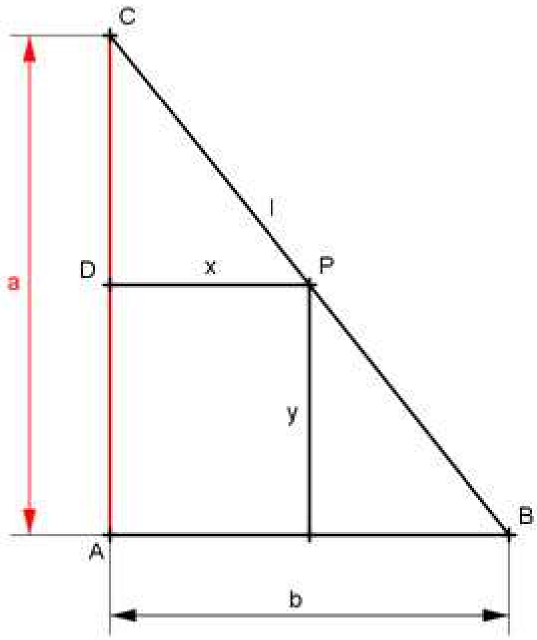
Extrem Aufgabe 119

Der Punkt P hat einen Abstand x vom Schenkel a und y vom Schenkel b des rechtwinkligen Dreiecks ABC. Wie lang ist der Schenkel a, wenn die Länge l der Strecke BC minimal sein soll?



Zielfunktion:

$$l^2 = a^2 + b^2$$

Nebenbedingung:

Strahlensatz:

$$\frac{b}{x} = \frac{a}{a - y} \quad | \cdot x$$

$$b = \frac{ax}{a - y}$$

$$b^2 = \frac{a^2x^2}{(a - y)^2}$$

In die Zielfunktion eingesetzt:

$$l^2_{(a)} = a^2 + \frac{a^2x^2}{(a - y)^2} \qquad 0 < a < \infty$$

$$l^2_{(a)} = \frac{a^2 * (a^2 - 2ay + y^2) + a^2x^2}{(a - y)^2}$$

$$l^2_{(a)} = \frac{a^4 - 2a^3y + a^2y^2 + a^2x^2}{(a - y)^2}$$

Quotientenregel:

$u' = 4a^3 - 6a^2y + 2ay^2 + 2ax^2$

$v' = 2 * (a - y)$

$$l^{2\prime}_{(a)} = \frac{(4a^3-6a^2y+2ay^2+2ax^2)*(a-y)^2 - 2*(a-y)*(a^4-2a^3y+a^2y^2+a^2x^2)}{(a - y)^4}$$

$$l^{2\prime}_{(a)} = \frac{(a - y)*[(4a^3-6a^2y+2ay^2+2ax^2)*(a-y) - 2*(a^4-2a^3y+a^2y^2+a^2x^2)]}{(a - y)^4}$$

$$l^{2\prime}_{(a)} = \frac{2a^4 - 2a^3y - 4a^3y + 6a^2y^2 - 2ay^3 - 2ayx^2}{(a - y)^3}$$

$$l^{2\prime}_{(a)} = \frac{2a^4 - 6a^3y + 6a^2y^2 - 2ay^3 - 2ayx^2}{(a - y)^3}$$

$$l^{2\prime}_{(a)} = \frac{2a * [(a^3 - 6a^2y + 6ay^2 - y^3) - yx^2]}{(a - y)^3}$$

$$l^{2\prime}_{(a)} = \frac{2a * [(a - y)^3 - yx^2]}{(a - y)^3}$$

$$\frac{2a * [(a - y)^3 - yx^2]}{(a - y)^3} = 0 \quad | *(a - y)^3$$

$2a * [(a - y)^3 - yx^2] = 0$

$2a = 0 \quad |:2$

$a = 0$ keine Lösung, außerhalb des Definitionsbereiches

$(a - y)^3 - yx^2 = 0 \quad |+yx^2$

$(a - y)^3 = yx^2 \quad |\sqrt[3]{}$

$a - y = \sqrt[3]{yx^2} \quad |+y$

$\mathbf{a = y + \sqrt[3]{yx^2}}$

$$b = \frac{(y + \sqrt[3]{yx^2}) * x}{y + \sqrt[3]{yx^2} - y} = \frac{xy + x * \sqrt[3]{yx^2}}{\sqrt[3]{yx^2}} = \frac{\sqrt[3]{x^3y^3}}{\sqrt[3]{yx^2}} + x$$

$b = \sqrt[3]{xy^2} + x$

oder

Summen- und Quotientenregel:

$$l^2{}_{(a)} = a^2 + \frac{a^2x^2}{(a - y)^2} \qquad 0 < a < \infty$$

$u' = 2ax^2$

$v' = 2 * (a - y)$

$$l^2{}'_{(a)} = 2a + \frac{2ax^2 * (a - y)^2 - 2 * (a - y) * a^2x^2}{(a - y)^4}$$

$$l^2{}'_{(a)} = 2a + \frac{(a - y) * [2ax^2 * (a - y) - 2 * a^2x^2]}{(a - y)^4}$$

$$l^2{}'_{(a)} = 2a + \frac{[2ax^2 * (a - y) - 2 * a^2x^2]}{(a - y)^3}$$

$$l^2{}'_{(a)} = \frac{2a * (a - y)^3 - 2ayx^2}{(a - y)^3}$$

usw. siehe oben.

Zur Beurteilung, ob $l^2{}''_{(a)}$ > oder < 0: (Begründung siehe Kurvendiskussion Aufgabe 105)

$$z = 2a * [(a - y)^3 - yx^2]$$

$$u' = 2$$

$$v' = 3 * (a - y)^2$$

$$z' = 2 * [(a - y)^3 - yx^2] + 3 * (a - y)^2 * 2a$$

$$I^{2''}_{(a)} = \frac{z'}{n} = \frac{2 * [(a - y)^3 - yx^2] + 3 * (a - y)^2 * 2a}{(a - y)^3}$$

$$I^{2''}_{(a)} = \frac{2 * (a - y)^3 - 2 * yx^2 + 6a * (a - y)^2}{(a - y)^3}$$

$$I^{2''}_{(a)} = \frac{(a - y)^2 * (2 * (a - y) + 6a) - 2yx^2}{(a - y)^3}$$

$$I^{2''}_{(a)} = \frac{2 * (a - y) + 6a}{(a - y)} - \frac{2yx^2}{(a - y)^3}$$

$$I^{2''}_{(a)} = \frac{8a - 2y}{(a - y)} - \frac{2yx^2}{(a - y)^3}$$

$$I^{2''}_{(y + \sqrt[3]{yx^2})} = \frac{8 * (y + \sqrt[3]{yx^2}) - 2y}{(y + \sqrt[3]{yx^2} - y)} - \frac{2yx^2}{(y + \sqrt[3]{yx^2} - y)^3}$$

$$I^{2''}_{(y + \sqrt[3]{yx^2})} = \frac{6y + 8 * \sqrt[3]{yx^2}}{\sqrt[3]{yx^2}} - \frac{2yx^2}{yx^2}$$

$$I^{2''}_{(y + \sqrt[3]{yx^2})} = \frac{6y + 8 * \sqrt[3]{yx^2}}{\sqrt[3]{yx^2}} - 2$$

$$I^{2''}_{(y + \sqrt[3]{yx^2})} = \frac{6y + 8 * \sqrt[3]{yx^2} - 2 * \sqrt[3]{yx^2}}{\sqrt[3]{yx^2}}$$

$$l^{2\prime\prime}_{(y + \sqrt[3]{yx^2})} = \frac{6y + 6 * \sqrt[3]{yx^2}}{\sqrt[3]{yx^2}} > 0 \text{ --> Minimum}$$

$$l^2(_{y + \sqrt[3]{yx^2})} = (y + \sqrt[3]{yx^2})^2 + (\sqrt[3]{xy^2} + x)^2$$

$$l^2(_{y + \sqrt[3]{yx^2})} = y^2 + 2y * \sqrt[3]{yx^2} + (\sqrt[3]{yx^2})^2 + x^2 + 2x * \sqrt[3]{xy^2} + (\sqrt[3]{xy^2})^2$$

$$l^2(_{y + \sqrt[3]{yx^2})} = x^2 + y^2 + 2y * \sqrt[3]{yx^2} + \sqrt[3]{y^2x^4} + 2x * \sqrt[3]{xy^2} + \sqrt[3]{x^2y^4}$$

$$l^2(_{y + \sqrt[3]{yx^2})} = x^2 + y^2 + 2y * \sqrt[3]{yx^2} + x * \sqrt[3]{xy^2} + 2x * \sqrt[3]{xy^2} + y * \sqrt[3]{x^2y}$$

$l^2(_{y + \sqrt[3]{yx^2})} = x^2 + y^2 + 3y * \sqrt[3]{yx^2} + 3x * \sqrt[3]{xy^2}$ absolutes Minimum, weil

wenn a --> o geht, dann b --> ∞ und umgekehrt -->

$$l^2_{(0)} = 0^2 + \infty^2 = \infty^2 > x^2 + y^2 + 3y * \sqrt[3]{yx^2} + 3x * \sqrt[3]{xy^2}$$

$$l^2_{(0)} = \infty^2 + 0^2 = \infty^2 > x^2 + y^2 + 3y * \sqrt[3]{yx^2} + 3x * \sqrt[3]{xy^2}$$