Integral Aufgabe 129

Berechnen Sie den Flächeninhalt A zwischen $f_{(x)} = \sqrt[3]{13x-12}$ und $g_{(x)} = x$.

Schnittpunkte:

$f_{(x)} = g_{(x)}$

$\sqrt[3]{13x-12} = x \mid ^3$

$13x - 12 = x^3 \mid -13x + 12$

$x^3 - 13x + 12 = 0$

Durch Probieren ermittelt $x_1 = 1$

Polynomdivision:

```
  x³ - 13x +12 : x - 1 = x² + x - 12
-(x³ - x²)
--------------
        x² - 13x
      -(x²  - x)
      ---------------
             -12x + 12
           -(-12x + 12)
            ------------
                     0
```

$x^2 + x - 12 = 0$

Linearfaktoren:

$x^2 + x - 12 = (x + 4)(x - 3) = 0$

$x_2 = -4$

$x_3 = 3$

$f_{(x)} - g_{(x)} = \sqrt[3]{13x-12} - x$

$A = \int_{-4}^{1}(\sqrt[3]{13x - 12} - x)dx + \int_{1}^{3}(\sqrt[3]{13x - 12} - x)dx$

Integration durch Substitution:

$u = 13x - 12$

$$u' = 13 \rightarrow dx = \frac{du}{13}$$

$$\int (\sqrt[3]{13x - 12})dx = \int \frac{u^{\frac{1}{3}}}{13} du$$

3

$$A = \int_{-4}^{1}(\sqrt[3]{13x - 12} - x)dx = \int_{-4}^{1} \frac{u^{\frac{1}{3}}}{13} du - \int_{-4}^{1} x \, dx + \int_{1}^{3} \frac{u^{\frac{1}{3}}}{13} du - \int_{1}^{3} x \, dx$$

$$A = \left|\frac{u^{4/3}}{13 * 4/3}\right|_{-4}^{1} - \left|\frac{x^2}{2}\right|_{-4}^{1} = \left|\frac{(13x - 12)^{4/3}}{13 * 4/3}\right|_{-4}^{1} - \left|\frac{x^2}{2}\right|_{-4}^{1} +$$

$$+ \left|\frac{(13x - 12)^{4/3}}{13 * 4/3}\right|_{1}^{3} - \left|\frac{x^2}{2}\right|_{1}^{3}$$

$$A = |0{,}058 - (14{,}77)| - |0{,}5 - 8| + |4{,}67 - 0{,}058| - |4{,}5 - 0{,}5|$$

$$A = |7{,}82|$$

A = 7,82