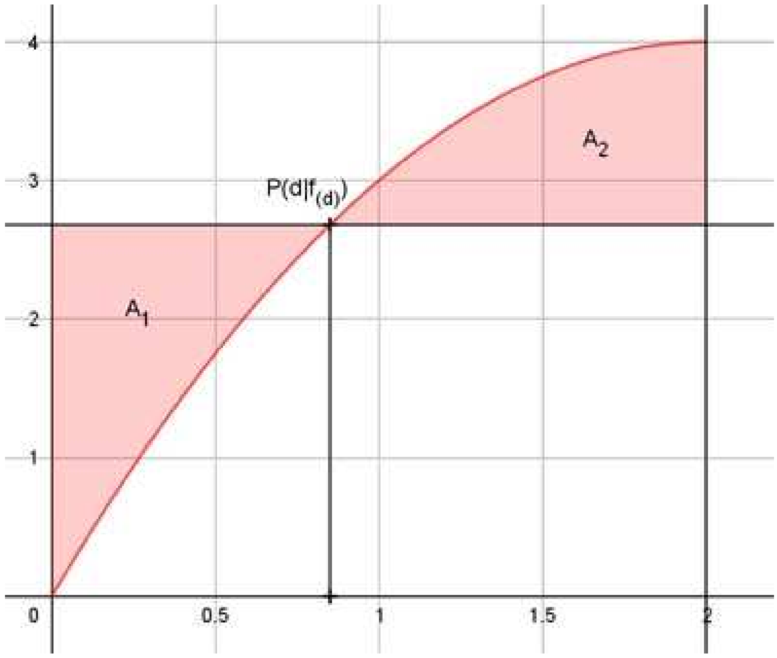
Integral Aufgabe 205

In welchem Abstand a muss eine Parallele g zur x-Achse verlaufen, damit sie den Graphen von $f_{(x)} = 4x - x^2$ zwischen $x = 0$ und $x = 2$ so teilt, dass die Flächen zwischen dem Graphen und g gleich groß sind?



$A_1 = A_2$

A_1 = Rechteck $d * f_{(d)} - \int_0^d (4x - x^2)dx$

$A_2 = \int_d^2 (4x - x^2)dx - (2 - d) * f_{(d)}$

Mit $f_{(d)} = 4d - d^2$:

$$d * (4d - d^2) - \int_0^d (4x - x^2)dx = \int_d^2 (4x - x^2)dx - (2 - d) * (4d - d^2)$$

$$4d^2 - d^3 - \left(2x^2 - \frac{x^3}{3}\right)\Big|_0^d = \left(2x^2 - \frac{x^3}{3}\right)\Big|_d^2 - (8d - 2d^2 - 4d^2 + d^3)$$

$$4d^2 - d^3 - 2d^2 + \frac{d^3}{3} = 8 - \frac{8}{3} - \left(2d^2 - \frac{d^3}{3}\right) - 8d + 6d^2 - d^3 \quad |+d^3$$

$$2d^2 + \frac{d^3}{3} = \frac{16}{3} - 2d^2 + \frac{d^3}{3} - 8d + 6d^2 \quad |- \frac{d^3}{3} - 2d^2$$

$$2d^2 - 8d + \frac{16}{3} = 0 \quad |:2$$

$$d^2 - 4d + \frac{8}{3} = 0$$

p, q - Formel:

$$p = -4,\ q = \frac{8}{3}$$

$$x_{1,2} = -\frac{-4}{2} \pm \sqrt{\left(\frac{-4}{2}\right)^2 - \frac{8}{3}}$$

$x_{1,2} = 2 \pm 1{,}15$

$x_1 = 3{,}15 > 2$ --> keine Lösung

$x_2 = 0{,}85$ --> $f_{(0,85)} =$ **a =** $4 * 0{,}85 - 0{,}85^2 =$ **2,68** gerundet

$A_1 = 0{,}85 * 2{,}68 - \int_0^{0,85}(4x - x^2)dx$

$$A_1 = 2{,}278 - \left|2x^2 - \frac{x^3}{3}\right|_0^{0,85} = 2{,}278 - (1{,}24)| = 1{,}04$$

$A_2 = \int_{0,85}^{2}(4x - x^2)dx - (2 - 0{,}85) * 2{,}68$

$$A_2 = \left|2x^2 - \frac{x^3}{3}\right|_{0,85}^{2} - 3{,}08 = |5{,}33 - (1{,}24)| - 3{,}08 = 1$$