Integral Aufgabe 259

Berechnen Sie den Flächeninhalt A, der von $f_{(x)} = \ln^2 x - 2 \ln x$ und der x-Achse begrenzt wird.

Nullstellen:

$\ln^2 x - 2 * \ln x = 0$

$\ln x(\ln x - 2) = 0$

$\ln x = 0$

$x = e^0$

$x_1 = 1$

$\ln x - 2 = 0 \quad |+2$

$\ln x = 2$

$x_2 = e^2$

$A = \int_1^{e^2} (\ln^2 x - 2 \ln x)dx = \int_1^{e^2} \ln^2 x \, dx - 2 * \int_1^{e^2} \ln x \, dx$

Partielle Integration für $\int \ln^2 x$:

$u = \ln^2 x,\ u' = \dfrac{2 * \ln x}{x},\ v' = 1,\ v = x$

$\int \ln^2 x = x * \ln^2 x - \int (\frac{2 * \ln x}{x}) * x \, dx$

$\int \ln^2 x = x * \ln^2 x - 2 * \int \ln x \, dx$

Partielle Integration für $\int \ln x \, dx$

$u = \ln x,\ u' = \dfrac{1}{x},\ v' = 1,\ v = x$

$\int \ln x \, dx = \ln x * x - \int \frac{1}{x} * x \, dx = \ln x * x - x$

$\int \ln^2 x = x * \ln^2 x - 2 * (\ln x * x - x)$

$$A = \int_1^{e^2} (\ln^2 x - 2\ln x)dx = \Big|x * \ln^2 x - 2(x * \ln x - x) - 2(x * \ln x - x)\Big|_1^{e^2}$$

$$A = \Big|x * \ln^2 x - 4(x * \ln x - x)\Big|_1^{e^2}$$

A = $|e^2 * 2^2 - 8e^2 + 4e^2 - (4)| =$ **4**