

Integral Aufgabe 61

Berechnen Sie den Flächeninhalt A zwischen dem Graphen von $f(x)$ und der x-Achse.

$$f(x) = (\ln x)^2 - 4$$

Nullstellen:

3. Binom:

$$(\ln x)^2 - 4 = (\ln x - 2)(\ln x + 2)$$

$$\ln x - 2 = 0 \quad | +2$$

$$\ln x = 2$$

$$x_1 = e^2 = 7,39$$

$$\ln x + 2 = 0 \quad | -2$$

$$\ln x = -2$$

$$x = e^{-2} = 0,135$$

$$\int ((\ln x)^2 - 4) dx = \int (\ln x)^2 dx - \int 4 dx$$

Partielle Integration:

$$u = (\ln x)^2, \quad u' = \frac{2 * \ln x}{x}, \quad v' = 1, \quad v = x$$

$$\int (\ln x)^2 dx = x * (\ln x)^2 - \int (2 * \ln x) dx$$

Partielle Integration:

$$\int (2 * \ln x) dx$$

$$u = \ln x, \quad u' = \frac{1}{x}, \quad v' = 2, \quad v = 2x$$

$$\int (2 * \ln x) dx = 2x * \ln x - \int \left(\frac{1}{x} * 2x\right) dx = 2x * \ln x - 2x = 2x * (\ln x - 1)$$

$$\int (\ln x)^2 dx = x * (\ln x)^2 - 2x * (\ln x - 1)$$

$$\int (\ln x)^2 dx - \int 4 dx = x * (\ln x)^2 - 2x * (\ln x - 1) - 4x$$

$$\int (\ln x)^2 dx - \int 4 dx = x * (\ln x)^2 - 2x * \ln x - 2x$$

$$A = | x * (\ln x)^2 - 2x * \ln x - 2x |_{0,135}^{7,39}$$

$$A = | 7,39 * (\ln 7,39)^2 - 2 * 7,39 * \ln 7,39 - 2 * 7,39 - (0,135 * (\ln 0,135)^2 - 2 * 0,135 * \ln 0,135 - 2 * 0,135) |$$

$$A = | -14,78 - 0,81 |$$

$$A = 15,59$$

