Kurven Aufgabe 109

$$\mathbf{f_{(x)} = \frac{x^3}{x - 1}}$$

Quotientenregel erste Ableitung:

$u = x^3$, $u' = 3x^2$

$v = x - 1$, $v' = 1$

$$f'_{(x)} = \frac{3x^2 * (x - 1) - 1 * x^3}{(x - 1)^2} = \frac{3x^3 - 3x^2 - x^3}{(x - 1)^2} = \frac{2x^3 - 3x^2}{(x - 1)^2}$$

Quotientenregel zweite Ableitung:

$u = 2x^3 - 3x^2$, $u' = 6x^2 - 6x$

$v = (x - 1)^2$

Kettenregel:

$v' = 2 * (x - 1)$

$$f''_{(x)} = \frac{(6x^2 - 6x)(x - 1)^2 - 2\,(x - 1) * (2x^3 - 3x^2)}{(x - 1)^4}$$

$$f''_{(x)} = \frac{(x - 1) * [6x^2 - 6x)(x - 1) - 2 * (2x^3 - 3x^2)]}{(x - 1)^4}$$

$$f''_{(x)} = \frac{6x^3 - 12x^2 + 6x - 4x^3 + 6x^2}{(x - 1)^3} = \frac{2x^3 - 6x^2 + 6x}{(x - 1)^3}$$

Zur Beurteilung, ob $f'''_{(x)} \neq 0$: (Begründung siehe Aufgabe 105)

$u = 2x^3 - 6x^2 + 6x$, $u' = 6x^2 - 12x + 6$

$$f'''_{(x)} = \frac{u'}{v} = \frac{6x^2 - 12x + 6}{(x - 1)^3} = \frac{6 * (x^2 - 2x + 1)}{(x - 1)^3} = \frac{6 * (x - 1)^2}{(x - 1)^3} = \frac{6}{(x - 1)}$$

--> $f'''_{(x)} \neq 0$ für alle $x \neq 1$

Definitionsbereich: $\mathbf{- \infty < x < \infty, x \neq 1}$

Wertebereich: $\mathbf{- \infty < f_{(x)} < \infty}$

Symmetrie: -

Asymptoten:

$(x - 1) = 0 \mid + 1$

$x = 1$

$$\mathbf{y =} x^3 : (x - 1) = \mathbf{x^2 + x + 1} + \frac{1}{x - 1}$$

$$\begin{array}{l}
-(x^3 - x^2) \\
\hline
\quad x^2 \\
\quad -(x^2 - x) \\
\hline
\qquad x \\
\qquad -(x - 1) \\
\hline
\qquad\quad 1
\end{array}$$

Nullstellen:

$x^3 = 0 \mid \sqrt[3]{}$

$x_{1,2,3} = 0$

Schnittpunkt mit der y-Achse:

$$f_{(0)} = \frac{0^3}{0 - 1} = 0$$

Sy (0|0)

Extrempunkte:

$2x^3 - 3x^2 = 0$

$x^2 * (2x - 3) = 0$

$x_{1,2} = 0;\ f(_{x1,2}) = 0$

$2x - 3 = 0 \mid +3$

$2x = 3 \mid :2$

$$x_3 = 1{,}5;\ f_{(x3)} = \frac{1{,}5^3}{1{,}5 - 1} = 6{,}75$$

$$f''_{(0)} = \frac{2 * 0^3 - 6 * 0^2 + 6 * 0}{(0 - 1)^3} = 0$$

$$f''_{(1,5)} = \frac{2 * 1,5^3 - 6 * 1,5^2 + 6 * 1,5}{(1,5 - 1)^3} = \frac{2,25}{0,5} > 0 \text{ -->}$$

Tiefpunkt (1,5|6,75)

Wendepunkte:

$2x^3 - 6x^2 + 6x = 0$

$2x * (x^2 - 3x + 3) = 0$

$2x = 0 \mid :2$

$x_1 = 0$; $f'_{(0)} = 0$, $f''_{(0)} = 0$, $f'''_{(0)} \neq 0$ -->

Wendepunkt (Sattelpunkt) bei (0|0)

$x^2 - 3x + 3 = 0$

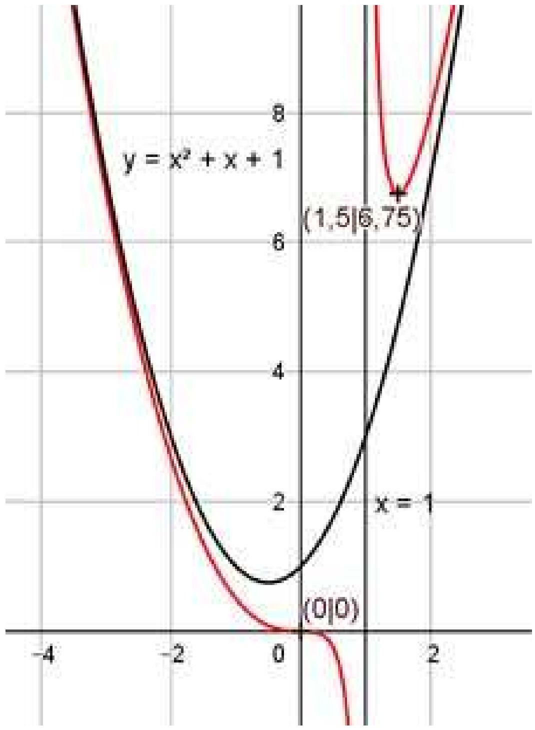
p, q - Formel

p = - 3, q = 3

$$x_{2,3} = \frac{-(-3)}{2} \pm \sqrt{\left(\frac{-3}{2}\right)^2 - 3}$$

$x_{2,3} = 1,5 \pm \sqrt{-0,75}$ --> keine Lösung

Graph:



8
y = x² + x + 1
(1,5|6,75)
6
4
2
x = 1
(0|0)
-4
-2
0
2