Kurven Aufgabe 181

$$\mathbf{f_{(x)} = \frac{3 * \cos x}{2 + \sin x}}$$ x im Bogenmaß

Quotientenregel erste Ableitung:

$u = 3 * \cos x$, $u' = - 3 * \sin x$

$v = 2 + \sin x$, $v' = \cos x$

$$f'_{(x)} = \frac{- 3 * \sin x * (2 + \sin x) - \cos x * 3 * \cos x}{(2 + \sin x)^2}$$

$$f'_{(x)} = \frac{- 6 * \sin x - 3 \sin^2 x - 3 \cos^2 x}{(2 + \sin x)^2} = \frac{- 6 * \sin x - 3 * (\cos^2 x + \sin^2 x)}{(2 + \sin x)^2}$$

$$f'_{(x)} = \frac{- 6 * \sin x - 3}{(2 + \sin x)^2}$$

Quotientenregel und Kettenregel zweite Ableitung:

$u = - 6 * \sin x - 3$, $u' = - 6 * \cos x$

$v = (2 + \sin x)^2$, $v' = 2 * (2 + \sin x) * \cos x = 2 * \cos x * (2 + \sin x)$

$$f''_{(x)} = \frac{- 6 * \cos x * (2 + \sin x)^2 - 2 * \cos x * (2 + \sin x) * (- 6 * \sin x - 3)}{(2 + \sin x)^4}$$

$$f''_{(x)} = \frac{(2 + \sin x)*[-12*\cos x - 6*\cos x*\sin x + 12*\cos x*\sin x + 6*\cos x]}{(2 + \sin x)^4}$$

$$f''_{(x)} = \frac{- 6 * \cos x + 6 * \sin x * \cos x}{(2 + \sin x)^3} = \frac{- 6 * \cos x * (1 - \sin x)}{(2 + \sin x)^3}$$

Zur Beurteilung, ob $f'''_{(x)} \neq 0$:

$u = - 6 * \cos x * (1 - \sin x)$

Produktregel:

$u' = 6 * \sin x * (1 - \sin x) - \cos x * (- 6 * \cos x)$

$u' = 6 \sin x - 6 \sin^2 x + 6 \cos^2 x$

$$f'''_{(x)} = \frac{u'}{v} = \frac{6 \sin x - 6 \sin^2 x + 6 \cos^2 x}{(2 + \sin x)^3} \text{ ist } \neq 0 \text{ für alle } x$$

Definitionsbereich: **$0 \leq x \leq 2\pi$**

Wertebereich:

$- 1{,}73 \leq f_{(x)} \leq 1{,}73$ (siehe Extrempunkte)

Nullstellen:

$$\frac{3 * \cos x}{2 + \sin x} = 0 \mid *(2 + \sin x)$$

$3 * \cos x = 0 \mid :3$

$\cos x = 0$

$x_1 = \pi/2 = 1{,}57 \triangleq 90°$ **$N_1\ (1{,}57|0)$**

$x_2 = (3/2)\pi = 4{,}71 \triangleq 270°$ **$N_2\ (4{,}71|0)$**

Schnittpunkt mit der y-Achse:

$$f_{(0)} = \frac{3 * \cos 0}{2 + \sin 0} = 1{,}5$$

$S_y\ (1{,}5|0)$

Extrempunkte:

$$\frac{- 6 * \sin x - 3}{(2 + \sin x)^2} = 0 \mid * (2 + \sin x)^2$$

$- 6 * \sin x - 3 = 0 \mid +3$

$- 6 * \sin x = 3 \mid :(-6)$

$\sin x = - 0{,}5$

$$x_1 = (7/6)\pi = 3{,}67 \triangleq 210°,\ f_{(3,67)} = \frac{3 * \cos 3{,}67}{2 + \sin 3{,}67} = - 1{,}73$$

$$f''_{(3,67)} = \frac{-6 * \cos 3{,}67 * (1 - \sin 3{,}67)}{(2 + \sin 3{,}67)^3} > 0 \text{ --> } $$ **Tiefpunkt (3,67|-1,73)**

$$x_2 = (11/6)\pi = 5{,}76 \mathrel{\hat{=}} 330°,\ f_{(5,24)} = \frac{3 * \cos 5{,}76}{2 + \sin 5{,}76} = 1{,}73$$

$$f''_{(5,76)} = \frac{-6 * \cos 5{,}76 * (1 - \sin 5{,}76)}{(2 + \sin 5{,}76)^3} < 0 \text{ --> } $$ **Hochpunkt (5,76|1,73)**

Wendepunkte:

$$\frac{-6 * \cos x * (1 - \sin x)}{(2 + \sin x)^3} = 0 \mid * (2 + \sin x)^3$$

$-6 * \cos x * (1 - \sin x) = 0$

$-6 * \cos x = 0 \mid : -6$

$\cos x = 0$

$x_1 = \pi/2 = 1{,}57 \mathrel{\hat{=}} 90°$ **WP_1 (1,57|0)**

$x_2 = (3/2)\pi = 4{,}71 \mathrel{\hat{=}} 270°$ **WP_2 (4,71|0)**

$1 - \sin x = 0 \mid +\sin x$

$\sin x = 0$ --> $x_1 = \pi/2$, $x_2 = (3/2)\pi$

Graph: