Kurven Aufgabe 56

$\mathbf{f_{(x)} = (x + 1)^2 * (x - 2)^2}$

$\mathbf{f_{(x)} = (x^2 + 2x + 1) * (x^2 - 4x + 4)}$

$\mathbf{f_{(x)} = x^4 - 2x^3 - 3x^2 + 4x + 4}$

$f'_{(x)} = 4x^3 - 6x^2 - 6x + 4$, $f''_{(x)} = 12x^2 - 12x - 6$, $f'''_{(x)} = 24x - 12$

Definitionsbereich: $\mathbf{- \infty < x < \infty}$

Wertebereich: $\mathbf{0 \leq f_{(x)} < \infty}$ (siehe Extrempunkte)

Asymptoten: -

Symmetrie: -

Nullstellen:

$x^4 - 2x^3 - 3x^2 + 4x + 4 = 0$

Durch Probieren gefunden $x_1 = 2$

Hornerschema:

	1	-2	-3	4	4
$x_1 = 2$		2	0	-6	-4
	1	0	-3	-2	0

Polynomdivision:

$$
\begin{array}{l}
\quad x^4 - 2x^3 - 3x^2 + 4x + 4 : (x - 2) = x^3 - 3x - 2 \\
-(x^4 - 2x^3) \\
\hline
\qquad\qquad 0 - 3x^2 + 4x + 4 \\
\qquad\qquad -(-3x^2 + 6x) \\
\hline
\qquad\qquad\qquad\qquad -2x + 4 \\
\qquad\qquad\qquad\quad -(-2x + 4) \\
\hline
\qquad\qquad\qquad\qquad\qquad 0
\end{array}
$$

$x^3 - 3x - 2 = 0$

Durch Probieren gefunden $x_2 = 2$

	1	0	-3	-2
$x_2 = 2$		2	4	2

$$\begin{array}{cccc} 1 & 2 & 1 & 0 \end{array}$$

Polynomdivision:

$$\begin{array}{l}
\quad x^3 - 3x - 2 : (x - 2) = x^2 + 2x + 1 \\
-(x^3 - 2x^2) \\
\text{------------} \\
\qquad 2x^2 - 3x - 2 \\
\quad -(2x^2 - 4x) \\
\quad \text{---------------} \\
\qquad\qquad x - 2 \\
\qquad\quad -(x - 2) \\
\qquad\quad \text{---------} \\
\qquad\qquad\quad 0
\end{array}$$

Doppelte Nullstelle bei 2 --> Berührpunkt, Extrempunkt

$x^2 + 2x + 1 = 0$

p, q - Formel:

$p = 2,\ q = 1$

$x_{3,4} = \frac{-2}{2} \pm \sqrt{\left(\frac{2}{2}\right)^2 - 1}$

$x_{3,4} = -1 \pm \sqrt{0}$

$x_{3,4} = -1$ (doppelte Nullstelle, Berührpunkt, Extrempunkt)

$N_{1,2}$ (2|0), $N_{3,4}$ (-1|0)

Schnittpunkt mit der y-Achse:

$f_{(0)} = 0^4 - 2 * 0^3 - 3 * 0^2 + 4 * 0 + 4 = 4$

S_y(0|4)

Extrempunkte: liegen bei 2 und -1

$4x^3 - 6x^2 - 6x + 4 = 0$

Hornerschema:

$$\begin{array}{l|cccc}
 & 4 & -6 & -6 & 4 \\
x_1 = 2 & & 8 & 4 & -4 \\ \hline
 & 4 & 2 & -2 & 0 \\
x_2 = -1 & & -4 & 2 & \\ \hline
 & 4 & -2 & 0 &
\end{array}$$

$4x - 2 = 0 \mid +2$

$4x = 2 \mid :4$

$x_3 = 0{,}5$, $f_{(0{,}5)} = 0{,}5^4 - 2 * 0{,}5^3 - 3 * 0{,}5^2 + 4 * 0{,}5 + 4 = 5{,}06$

Polynomdivision: $x_1 = 2$

$$
\begin{array}{l}
\ \ 4x^3 - 6x^2 - 6x + 4 : (x - 2) = 4x^2 + 2x - 2 \\
-(x^3 - \ 8x^2) \\
\text{------------} \\
\qquad\qquad 2x^2 - 6x + 4 \\
\qquad\quad -(2x^2 - \ 4x) \\
\qquad\qquad \text{---------------} \\
\qquad\qquad\qquad\quad -2x + 4 \\
\qquad\qquad\qquad -(-2x + 4) \\
\qquad\qquad\qquad\quad \text{-----------} \\
\qquad\qquad\qquad\qquad\quad\ 0
\end{array}
$$

Polynomdivision: $x_2 = - 1$

$$
\begin{array}{l}
\ \ 4x^2 + 2x - 2 : (x + 1) = 4x - 2 \\
-(4x^2 + 4x) \\
\text{------------} \\
\qquad\quad -2x - 2 \\
\qquad -(-2x - 2) \\
\qquad \text{-----------} \\
\qquad\qquad\quad 0
\end{array}
$$

$f''_{(0{,}5)} = 12 * 0{,}5^2 - 12 * 0{,}5 - 6 < 0$ --> **Hochpunkt (0,5|5,06)**

$f''_{(2)} = 12 * 2^2$ +- $12 * 2 - 6 > 0$ --> **Tiefpunkt (2|0)**

$f''_{(-1)} = 12 * (-1)^2 - 12 * (-1) - 6 > 0$ --> **Tiefpunkt (-1|0)**

Wendepunkt:

$12x^2 - 12x - 6 = 0$

A, B, C - Formel:

A = 12, B = - 12, C = - 6

$$x_{1,2} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4*12*(-6)}}{2 * 12} = \frac{12 \pm \sqrt{432}}{24}$$

$$x_{1,2} = \frac{12 \pm 20{,}8}{24}$$

$x_1 = 1,37$, $f_{(1,37)} = 1,37^4 - 2 * 1,37^3 - 3 * 1,37^2 + 4 * 1,37 + 4 = 2,25$

$x_2 = - 0,37$

$f_{(-0,37)} = (-0,37)^4 - 2 * (-0,37)^3 - 3 * (-0,37)^2 + 4 * (-0,37) + 4 = 2,25$

$f'''_{(1,37)} = 24 * 1,37 - 12 \neq 0$ --> **Wendepunkt (1,37|2,25)**

$f'''_{(-0,37)} = 24 * (-0,37) - 12 \neq 0$ --> **Wendepunkt (-0,37|2,25)**

Graph: